

# Math 110. Some Review Materials for Test 1

Test 1 will be on Thursday, February 25. Bring a calculator if you have one.

1. To prepare for the exam, review all your class notes and chapters 1 and 2 through what we cover on Tuesday (beginning of Section 2.5) which is both easy and fun.
2. Be sure to justify all your work. HOW you (try to) solve a problem may be as important as your final answer.
3. Avoid crossing out. Use a pencil and eraser. Neatness is a plus!
4. Remember that your **journal** is due at the exam. Double check that you have tried all the journal problems listed online. Remember this journal should contain only journal problems. Follow the journal guidelines on the syllabus (highlight problem numbers, etc.)

**Types of problems you may encounter on the exam:** For all of these problems it will be helpful if you can explain the process you are using.

- a) Solve a simple logic problem of the type in Chapter 1.
- b) Use the Pigeonhole Principle to solve a problem. You should be able to explain your answer in terms of ‘objects’ and ‘containers’ and you should use the appropriate formula,  $\lceil \frac{m}{n} \rceil$ , and know what it means.
- c) Discuss various aspects of the Fibonacci numbers and apply your knowledge of them and similar sequences of numbers. Can you list them? Can you write a formula that tells how to generate them and do you know what the terms in that formula mean?
- d) What is the Golden ratio? Can you describe how we found it? Where else does it occur? What is its approximate value? What is its exact value?
- e) Be able to work with infinite expressions such as

$$\sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \dots}}}}}$$

$$w = 2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \ddots}}}}}$$

- f) Play a game of Fibonacci Nim (make sure you can describe what the next move is in a winning strategy).
- g) Know the Division Algorithm and the Prime Factorization Theorem.
- h) The definition of a prime number, non-prime number, even number, odd number, twin primes.
- i) Using the Sieve of Eratosthenes and explaining why it works.
- j) Can you explain what a conjecture is? Do you know a famous example of one?
- k) Perform modular (clock) arithmetic.
- l) Check whether a UPC symbol is correct and/or find a missing digit in such a symbol. (You should know the formula for the check digit!)
- m) Applying modular arithmetic to other sorts of ‘product codes’ (whose formulas I would give you, e.g., bank routing number).
- n) Caesar ciphers (which are easy and fun) if we get that far on Tuesday.
- o) Fermat’s Little Theorem (not his last) if we get this far.
- p) Any problem that is like a journal problem. (I expect to ask one or two such problems).

## Some practice exercises

Come prepared to discuss these on Tuesday with your fellow group members.

1. There are 2000 HWS students. Must there be six with the same birth date? Carefully explain. Must there be at least one date on which at most five students were born? Explain.
2. a) You are playing Fibonacci Nim. You start with 180 sticks. What is your first move? Explain the winning strategy.  
b) Suppose you start with 89 sticks?
3. Today is Tuesday. What day of the week will it be 1712 days from now?
4. Determine

a)  $6(14)^6 \pmod{12}$       b)  $85 + (27)(51) \pmod{9}$       c)  $(13)^{100} \pmod{12}$       d)  $5^{345} \pmod{12}$   
Hint for part (d): Use  $5^{345} = 5^{(2 \times 172) + 1}$

5. Discuss a journal problem. Do your group members agree on a solution?
6. Solve for

$$w = \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \dots}}}}}$$

Explain how you did it.

7. Explain why  $(2 \times 3 \times 4 \times \dots \times 50) + 1$  does not have any factors less than or equal to 50.
8. The Goldbach conjecture says that every even number greater than 2 can be written as the sum of two primes. No one knows if it is true. What about the odd Goldbach conjecture? Can every odd number greater than 3 be written as the sum of two prime numbers? If so, prove it; if not, find the smallest counterexample and show that the number given is definitely not the sum of two primes. Try some!