Math 331: Day 3

Reading, Practice, and Journal Work

This is material we will be working on today and the next few days.

1. Reread Section 1.2 and review partially ordered sets from Math 135, as needed. Here are a few problems in Section 1.2 that you will be prepared to try at this point.

(a) Problem 1.2.1. This should be familiar from Math 135. Try Problem 1.2.11. Again find l.u.b.'s.

(b) Problem 1.2.3. Another a simple exercise with the definition of lub.

(c) Problem 1.2.4.

2. Cut basics: Which of the following sets are cuts? (Prove or disprove.)

(a) $\{q \in \mathbb{Q} : q < 5\}$ (b) $\{q \in \mathbb{Q} : q^2 < 5\}$ (c) $\{q \in \mathbb{Q} : q^3 < 5\}$ (d) $A = \{q \in \mathbb{Q} : q > 5\}$ (e) A^c

3. Let α and β be cuts.

(a) Prove that $\alpha \cup \beta$ is a cut.

(b) Prove or disprove: $\alpha \cap \beta$ is a cut.

(c) Prove or disprove: α^C is a cut.

4. Prove: **Theorem:** *Let S be a subset of $\mathbb{R}$. λ is the least upper bound of S if and only if (i) $\lambda \geq \alpha$ for all $\alpha \in S$; and (ii) for any $\beta < \lambda$, there is a $\theta \in S$ such that $\beta < \theta$.*

5. ☞ Problem 1.2.2. This is sure to be on the **next assignment**. Try proving this result for any partially ordered set $(A, \leq)$.

6. (a) Let $(A, \leq)$ be a partially ordered set and let $S \subseteq A$. Complete the following definition: *An element $\beta \in A$ is **lower bound** for S if*

(b) Let $(A, \leq)$ be a partially ordered set and let $S \subseteq A$. Define the term **greatest lower bound** of S.

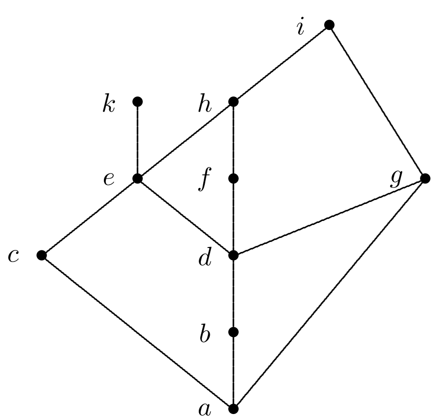
(c) List some finite and infinite subsets of $\mathbb{R}$. Which have lower bounds? Which have greatest lower bounds? Do all subsets of $\mathbb{R}$ have lower bounds? Greatest lower bounds? Is there a finite subset $S \subseteq \mathbb{R}$ which does not have a lower bound?

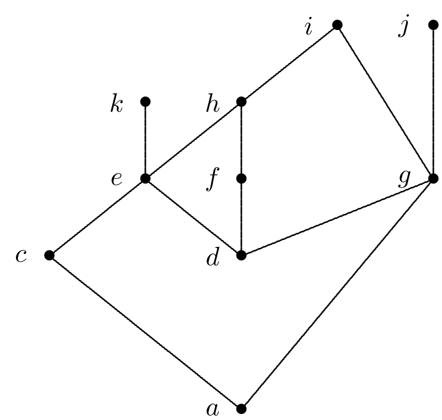
7. (a) Shortly we will discuss addition of cuts. Begin by rereading the definition of the **sum** of two cuts in the text. From Problem 6 from Day 2, the rational number 0 is the real number (cut) defined by $\mathbf{Zero} = \{q \in \mathbb{Q} \mid q < 0\}$. Prove: If α is any cut, then $\mathbf{Zero} + \alpha = \alpha$. Hint: Remember this is a set equality you are trying to prove. You must show $\mathbf{Zero} + \alpha \subset \alpha$ and $\alpha \subset \mathbf{Zero} + \alpha$.

(b) From Problem 6 from Day 2, the rational numbers 1 and 2 are the real numbers (cuts) defined by $\mathbf{One} = \{q \in \mathbb{Q} \mid q < 1\}$ and $\mathbf{Two} = \{q \in \mathbb{Q} \mid q < 2\}$. Prove: $\mathbf{One} + \mathbf{One} = \mathbf{Two}$. Hint: Remember this is a set equality you are trying to prove.

8. In the diagrams below, one element is related to another if and only if an upward path can be found that connects them (even if it traverses other elements in between). And naturally an element is related to itself (not explicitly shown in the diagram). Find all of the upper bounds for each of these sets for each figure.

(a) $\{c, d\}$ (b) $\{a, d, g\}$ (c) $\{e, f, g\}$ (d) $\{k, g\}$ (e) $\{a, c, d\}$ (f) $\{a, d\}$





(g) Now find the least upper bounds for each of the sets above (if they exist).