Math 331: Day 8

Group Work Session Today at 3:00 in Lansing 3:10. Get help on homework!

Classwork and Journal Work

Reread Section 1.4. And begin Section 2.1 which should be very familiar.

1. Determine which of these sets are bounded. Which are bounded above and which are bounded below? If a set is bounded above, find its least upper bound. If it is bounded below, find its greatest lower bound.

a) $\mathbb{Z}$ **b)** $\mathbb{Q}$ **c)** $A = \{1/n^2 \mid n \in \mathbb{N}\}$ **d)** $B = (0, 1)$

e) $C = [0, 1]$ **f)** $D = \{1, 2, 3\}$ **g)** $X = \{1 - \dfrac{1}{n} \mid n \in \mathbb{N}\}$ **h)** $Y = [0, 1)$

2. Let $\{O_n\}_{n\in\mathbb{N}} = \{(\frac{1}{n}, 1 - \frac{1}{n}) \mid n \in \mathbb{N}\}$ and $\{P_n\}_{n\in\mathbb{N}} = \{(\frac{1}{n}, 1 + \frac{1}{n}) \mid n \in \mathbb{N}\}$.

a) Is $\{P_n\}$ an open cover of $[0, 1]$? Is there a finite subcover?

b) Explain why any open cover of D has a finite subcover.

c) $\{P_n\}$ covers $B = (0, 1)$. Is there a finite subcover?

d) Does $\{O_n\}$ cover $(0, 1)$? If so, is there a finite subcover?

e) Show that $\{U_n\}_{n\in\mathbb{N}} = \{(-n, n) \mid n \in \mathbb{N}\}$ covers $S = [\frac{1}{2}, \infty)$, but there is no finite subcover.

f) Does $\{Q_x\}_{x\in[0,2]} = \left\{(x - \frac{1}{3}, x + \frac{1}{3}) \mid x \in [0, 2]\right\}$ cover [0,2]? Is there a finite subcover?

3. Find sets that satisfy the following conditions.

a) A set S that has some accumulation points.

b) A set S that has an infinite number of accumulation points.

c) A set S whose points are its accumulation points.

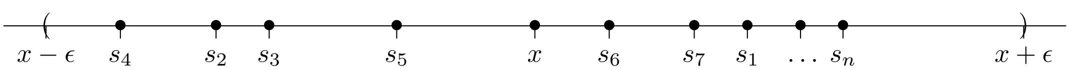
d) A set S with an accumulation point x not in S.

e) A set S whose only accumulation point(s) are not in S.

f) A set S some of whose points are accumulation points, but others are not.

g) A non-empty set S with no accumulation points.

h) Problem 1.4.9: What are the accumulation points of any finite set? Prove it. The following diagram may help you think about it.



4. Carefully prove that every real number x is an accumulation point of $\mathbb{Q}$.

5. Suppose that $\{O_\alpha\}$ is an open cover of the set C in Problem 1. Does it have a finite subcover? Explain.

6. A great problem once we get this far: Problem 1.4.15.

Hand In Wednesday: Problems from Section 1.4. Assigned Last Time.

9. Problem 1.4.1, but use the interval $[1, 5]$ instead.

10. Problem 1.4.2 (a and c). No proof, just make sure the covers work.

11. Problem 1.4.4 (a).

12. Problem 1.4.5

13. Problem 1.4.11. No need to show work. Just carefully read and use Definition 1.4.3

14. a) **EZ Bonus:** You may assume that the collection $\{O_n\}_{n\in\mathbb{N}} = \{(\frac{2}{n}, 2) \mid n \in \mathbb{N}\}$ is an open cover for the interval $(0, 1]$. Carefully prove that $\{O_\alpha\}_{\alpha\in A}$ has no finite subcover of $(0, 1]$.

b) Suppose that the collection $\{O_\alpha\}_{\alpha\in A}$ is another open cover for the interval $(0, 1]$. Suppose further that $0 \in \bigcup_{\alpha\in A} O_\alpha$. Prove that $(0, 1]$ has a finite subcover from $\{O_\alpha\}$.