Math 331: Day 9

Read Section 2.1 (including the problems) on functions, which should largely be review. We will not cover this in class. Read Section 2.2 on limits. Pay particular to Definition 2.2.1 (memorize) and Examples 2.2.1–6. **Be prepared** to discuss the questions below on Wednesday.

For Next Time

1. a) Complete the definition: If $S \subseteq \mathbb{R}$, then x is an **accumulation point** of S if:

b) Negate this definition: x is **not** accumulation point if:

2. From Math 135 (and 331), a **function** f is defined as relation be between two sets A (called the domain) and B (called the codomain) where

1. for each $a \in A$, there is a $b \in B$ so that $(a, b) \in f$, and
2. if (a, b) and (a, c) are both in f, then $b = c$.

If $A = \mathbb{R}$ and $B = \mathbb{R}$, then which of the following collections are functions.

a) $f = \{(a, b) \mid b = a^2\}$ **b)** $g = \{(a, b) \mid a = |b|\}$ **c)** $h = \{(a, b) \mid b = \tan a\}$

d) True or false: $f = \{(a, b) \mid b = \sin a\}$ is not a function because $2 \in B = \mathbb{R}$, but there is no $a \in A = \mathbb{R}$ so that $(a, 2) \in f$.

e) When using the 'usual' $y = f(x)$ function notation for a function (which we almost always will) what do we assume about the domain of f?

f) Describe the function $f(x) = \sqrt{x^2 + 1}$ as a collection of ordered pairs, as above.

g) What is the domain of $f \circ g$ if $f(x) = \sqrt{x - 4}$ and $g(x) = x^2$?

h) What is the domain of $g \circ f$ if $f(x) = \sqrt{x - 4}$ and $g(x) = x^2$?

i) Why do my calculus students think $f(x) = x + 1$ and $g(x) = \frac{x^2-1}{x-1}$ are the same function? Are they?

j) My calculus students think $f(x) = |x|$ and $g(x) = \sqrt{x^2}$ are not the same function? Are they?

3. Draw a graph of the Dirichlet function $D(x)$ on page 48.

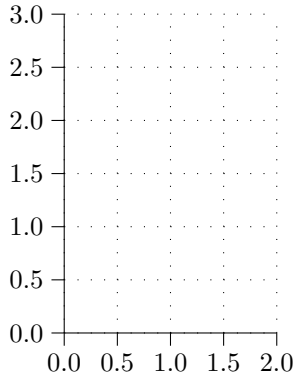
4. Which of the following are polynomials? Which are rational functions?

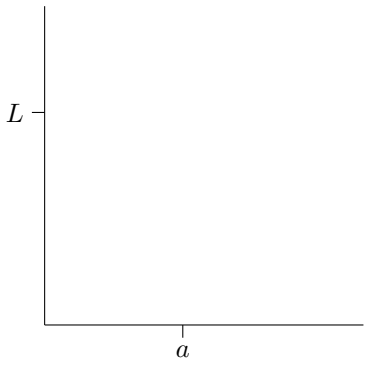
a) $p(x) = 6x^8 - \pi x^3 + 11$ **b)** $p(x) = 6x^8 - \pi x^{3/2} + 11$ **c)** $p(x) = 6x^{-8} - \pi x^3 + 11$

d) $\dfrac{x^2 - 1}{x + 1}$ **e)** 6 **f)** $\dfrac{2}{x} + \dfrac{x^2 - 3}{x^4 + 9}$

5. In Definition 2.2.1 of **limit**, what is the importance of the condition $0 < |x - a|$?

6. a) On the axes on the left below, draw (shade) the set of all points (x, y) that satisfies both of these conditions: $|x - 1| < \frac{1}{2}$ and $|y - 1.5| < 1$.





b) How would modify your answer if the first condition changed: $0 < |x - 1| < \frac{1}{2}$ and $|y - 1.5| < 1$.

c) On the axes on the right above, draw (shade) the set of all points (x, y) that satisfies both of these conditions: $0 < |x - a| < \delta$ and $|y - L| < \epsilon$. (Your choice of δ and ϵ.)

7. Self Check: Find sets that satisfy the following conditions.

a) A set S that has some accumulation points.

b) A set S that has an infinite number of accumulation points.

c) A set S whose points are its accumulation points.

d) A set S with an accumulation point x not in S.

e) A set S whose only accumulation point(s) are not in S.

f) A set S some of whose points are accumulation points, but others are not.

g) A non-empty set S with no accumulation points.