Math 331 Day 32: Examples of Sequences

1. a) Which of these sequences **converge**?(See the next page, too!)

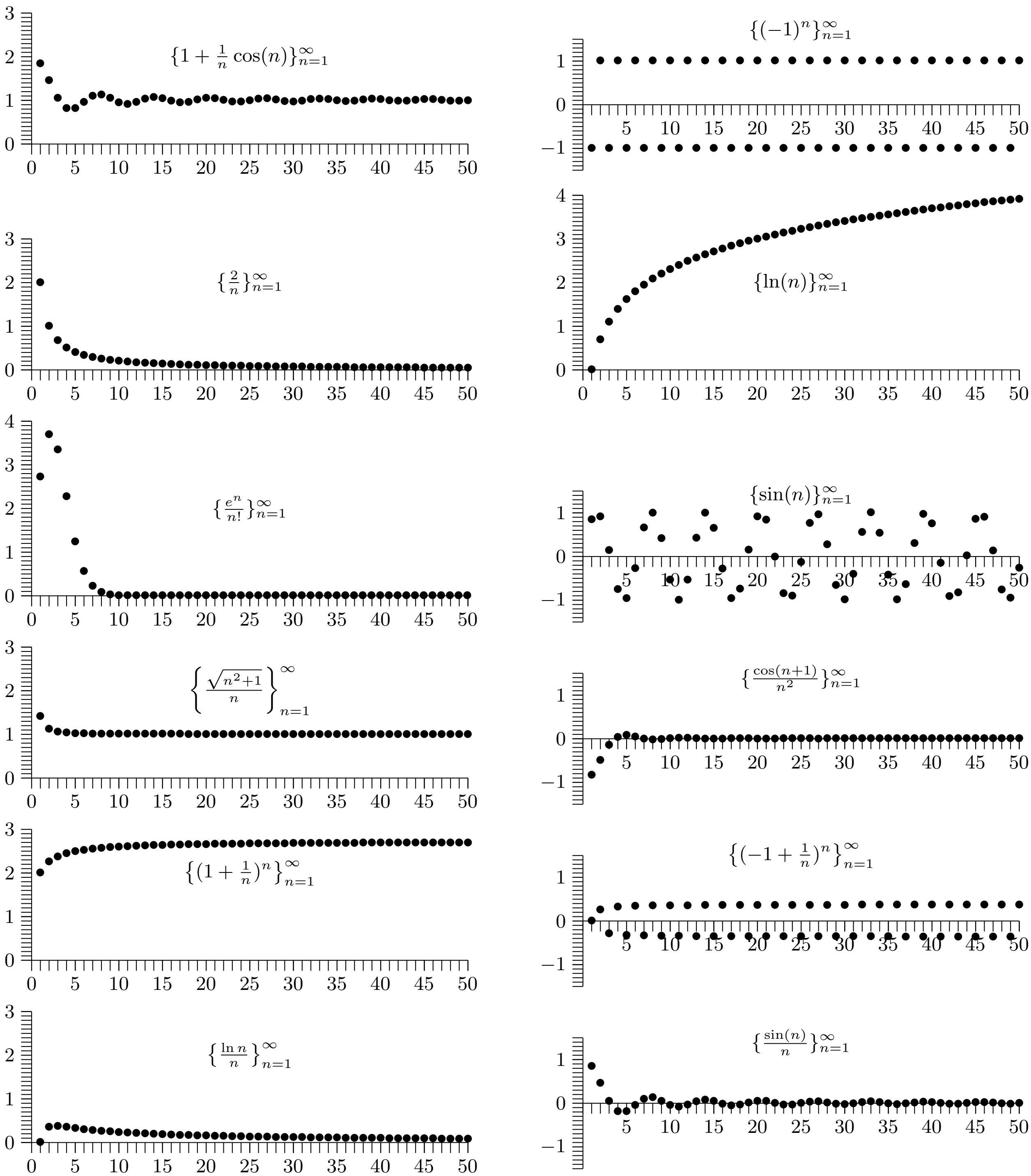
b) Which are **decreasing**?

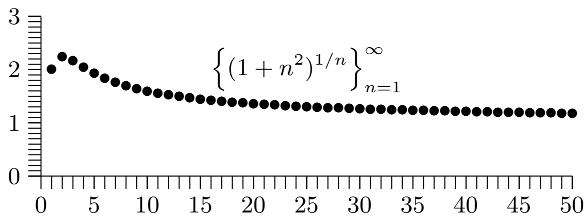
c) Which are **increasing**?

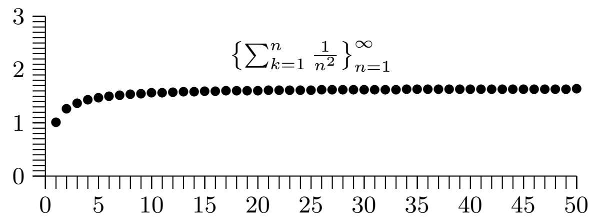
d) Which are **monotone**?

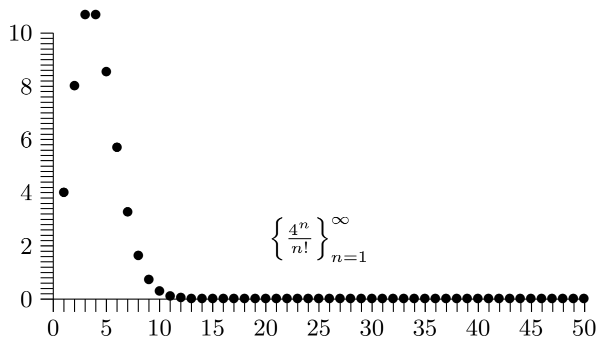
e) Which are monotone after the first few terms? A sequence is **eventually monotone** if there is an integer N so that $\{a_n\}_{n=N}^{\infty}$ is monotone.

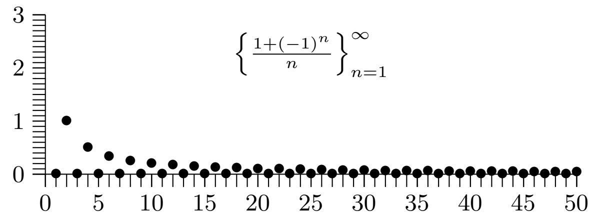
f) Which are **bounded**?











Day 32: Classwork/Practice on limits of sequences

1. Prove or give a counterexample:

a) If $\{s_n\}_{n=1}^{\infty}$ converges to s, then If $\{|s|_n\}_{n=1}^{\infty}$ converges to $|s|$.

b) If $\{|s_n|\}_{n=1}^{\infty}$ converges to $|s|$, then $\{s_n\}_{n=1}^{\infty}$ converges to s.

c) If $\{|s_n|\}_{n=1}^{\infty}$ converges to 0, then $\{s_n\}_{n=1}^{\infty}$ converges to 0.

2. Find an example:

a) a convergent sequence of rational numbers that converges to an irrational number

b) a convergent sequence of irrational numbers that converges to a rational number

3. Given a sequence $\{s_n\}_{n=1}^{\infty}$ and $k \in \mathbb{N}$. Let $\{t_n\}_{n=1}^{\infty}$ be the sequence defined by $t_n = s_{n+k}$. So the terms in the sequences are the same, it's just that $\{t_n\}_{n=1}^{\infty}$ has the first k terms cut off. Prove that $\{t_n\}_{n=1}^{\infty}$ converges if and only if $\{s_n\}_{n=1}^{\infty}$ converges, and if they converge then $\lim_{n\to\infty} t_n = \lim_{n\to\infty} s$. So the convergence of a sequence is not affected by omitting (or changing) a finite number of terms.

4. a) Suppose that $\lim_{n\to\infty} s_n = 0$. If t_n is a bounded sequence prove that $\lim_{n\to\infty} s_n t_n = 0$.

b) Show by an example that the boundedness of $\{t_n\}_{n=1}^{\infty}$ is a necessary condition in part (a).

5. Prove that $\{\ln n\}_{n=1}^{\infty}$ diverges.

6. a) Determine whether $\left\{\frac{\cos n}{n}\right\}_{n=1}^{\infty}$ converges. Prove your result.

b) Determine whether $\left\{\dfrac{5n^2 + \sqrt[3]{n} + \cos n}{2n^2 - 2n + 1}\right\}_{n=1}^{\infty}$ converges.

7. Determine whether $\left\{\frac{1}{2^n}\right\}_{n=1}^{\infty}$ converges. Prove your result.

8. For $n \geq 1$, define $a_n = 1 + \frac{1}{2} + \frac{1}{2^2} + \cdots + \frac{1}{2^n}$.

a) Show that $a_n = 2 - \frac{1}{2^{n+1}}$.

b) Show that $\{a_n\}_{n=1}^{\infty}$ converges to 2.