Math 331 Homework: Day 38

Read Section 4.5 on sequences of functions which we will begin next time.

Keep in Mind

1. Thinking about series tests:

a) nth term test (for divergence)—often not useful.

b) geometric series test—easy to spot when to use.

c) comparison test or limit comparison test—usually compare to a p-series or possibly a geometric series.

d) p-series—easy to spot when to apply; use with comparison test.

e) ratio test—especially useful with factorials and nth powers.

f) root test—useful with nth powers, but ratio test is often a better choice.

Part 2, of Hand In

Add these to the the problems previously assigned for Wednesday.

3. (Revised). Think about absolute convergence in Section 4.4. If you prove part (b), there is no need to do (a).

a) Prove: If $|r| < 1$, prove that $\sum_{k=1}^{\infty}(k+1)r^k$ converges. (Note: r may be negative.)

6. The following can be done very quickly. Prove this modification of the Ratio Test. *Suppose that* $\sum_{k=1}^{\infty} a_k$ *is a series such that* $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = r$. *If* $r < 1$, *then the series converges.*

7. Extra Credit This is both easy and interesting. Page 199 #4.4.6.

8. Extra Credit: Page 200 #4.4.7.

9. Extra Credit: Determine whether $\sum_{k=2}^{\infty}\ln(1-\frac{1}{n^2})$ converges or diverges.

Classwork

For this time and next:

1. On the current HW, you are proving the **Limit Comparison Test:** Suppose that $\sum_{k=1}^{\infty} a_k$ and $\sum_{k=1}^{\infty} b_k$ are both *positive* series and that

$$\lim_{n\to\infty}\frac{a_n}{b_n} = L > 0$$

(where L is finite). Then either both series converge or both diverge. *Note:* As with the Comparison test, the Limit comparison test can be modified to require only that a_n and b_n are positive from all n greater than some integer M.

2. Show: If $\sum_{k=1}^{\infty} a_k$ is a convergent series, then there exists an integer N such that if $n > N$, then $\left|\sum_{k=n+1}^{\infty} a_k\right| < \epsilon$. That is, the infinite tail of the series can be made arbitrarily small.

3. Determine (justify) whether these series converge or diverge. (Which p in (f) gives convergence?)

a) $\sum_{k=1}^{\infty}\frac{k!}{5^k}$ **b)** $\sum_{k=1}^{\infty}\frac{4^k}{5^k+3}$ **c)** $\sum_{k=1}^{\infty}\sin(\frac{1}{k})$ **d)** $\sum_{k=1}^{\infty}\frac{\sqrt[3]{k}}{2k-1}$ **e)** $\sum_{k=1}^{\infty}\frac{e^{2k}}{k^k}$ **f)** $\sum_{k=1}^{\infty}\frac{\ln k}{k^p}$

4. Determine whether these series converge or diverge.

a) $\sum_{k=1}^{\infty}\frac{k}{\sqrt{k^7+1}}$ **b)** $\sum_{k=1}^{\infty}\frac{(2k)!}{(k!)^2}$ **c)** $\sum_{k=1}^{\infty}\frac{k!}{2^k}$ **d)** $\sum_{k=1}^{\infty}\frac{3^k}{k^3}$

5. Can a negative series converge conditionally? Explain.