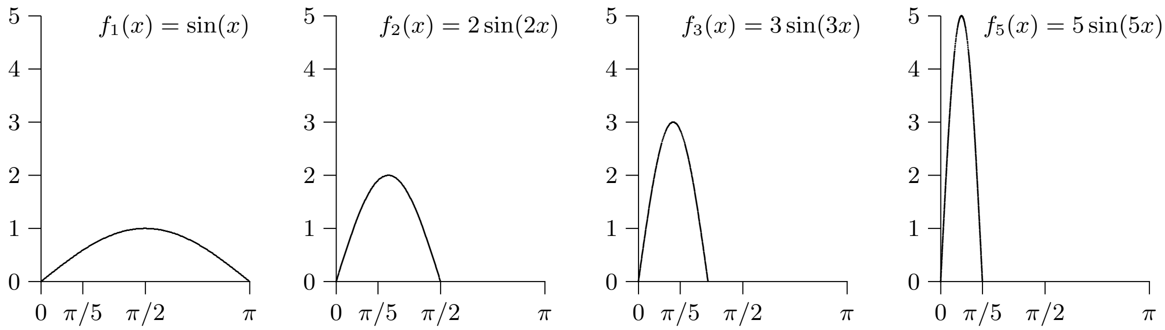
Math 331: Day 41

Read Section 4.5. Skim the first few pages of 4.6.

Problems

1. The sequence of functions $\{f_n(x)\}_{n=1}^{\infty}$ where $f_n(x) = \begin{cases} n\sin(nx), \text{if} \\ 0, \text{ if } \frac{\pi}{n} < x \leq \pi \end{cases}$



(*a*) Explain why $\lim_{n\to\infty} f_n(x) = 0$.

(*b*) Determine $\displaystyle\lim_{n\to\infty} \int_0^{\pi} f_n(x)\,dx$.

(*c*) Determine $\displaystyle\int_0^{\pi} \lim_{n\to\infty} f_n(x)\,dx$.

(*d*) Without doing a lot of work does $\{f_n(x)\}_{n=1}^{\infty}$ converge uniformly? Explain.

2. For $x \in [0,1]$ and $n \in \mathbb{N}$, define $f_n(x) = 2x + \frac{x}{n}$. Notice that f_n is differentiable, continuous, and integrable on $[0,1]$

(*a*) Find the pointwise limit function $f(x)$.

(*b*) Is $f(x)$ continuous?

(*c*) Is $f(x)$ differentiable and if so, is $f'(x) = \lim_{n\to\infty} f_n'(x)$?

(*d*) Is $f(x)$ integrable and if so, is $\int_0^1 f(x)\,dx = \lim_{n\to\infty} \int_0^1 f_n(x)\,dx$?

3. (*a*) Find the pointwise limit of the sequence of functions $f_n(x) = \dfrac{1}{1+x^{2n}}$ on $(-\infty, \infty)$.

(*b*) Is the convergence uniform? Explain your reasoning.

——————————— Think about this ———————————

4. Find the pointwise limit of the series $\sum_{k=0}^{\infty} x(1-x)^k$ on the interval $[0,1]$. Show that the convergence is not uniform. (See Example 4.6.2.) (That is, show the sequence of partial sums $\{S_n(x)\}_{n=1}^{\infty} = \{\sum_{k=0}^{n} f_k(x)\}_{n=1}^{\infty}$ converges pointwise but not uniformly).

5. Let $f_n(x) = \dfrac{x^n}{n!}$.

(*a*) Show $\displaystyle\sum_{k=0}^{\infty} f_k(x)$ converges pointwise but not uniformly on all of $\mathbb{R}$ (i.e., show the sequence of partial sums $\{S_n(x)\}_{n=1}^{\infty} = \{\sum_{k=0}^{n} f_k(x)\}_{n=1}^{\infty}$ converges point wise but not uniformly).

(*b*) Show $\displaystyle\sum_{k=0}^{\infty} f_k(x)$ converges uniformly on $[-s, s]$ for any $s \in \mathbb{R}$.